**Indian Institute of Technology Mandi**

**IC 110: B.Tech. I year**

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**Odd Semester 2012-2013**

**Tutorial-1 (Limit, Continuity and Differentiability)**

1. Suppose is a continuous function and , then show that  such that .
2. Let and is defined on. Consider



show that but not implies .

1. Let is continuous and defined on and if then is constant.
2. Let be a function and we say that is a fixed point of if. Suppose is differentiable and . Prove that has at most one fixed point.
3. If be a function such that , Prove that is constant.
4. Show that every polynomial of odd degree has at least one real root.
5. Suppose that  satisfies  for each  in ,  
   a) Show that  for all  in  and  in . (R = Reals and Z = Integers)  
   b) Prove that  is continuous at 0 if and only if  is continuous on .
6. Show that if is continuous on the closed interval , then there is a function g which is continuous on , and which satisfies for all in .
7. Let be a continuous function. Then prove that range of  is a closed and bounded interval. What will happen if domain of given function be an open interval instead of a closed Interval.
8. Prove that if is continuous at ,thenis continuous at . Is the converse true?
9. Construct a function which is discontinuous everywhere except at 10 points.
10. Let be continuous on [a,b] and let when is rational. Prove that for every .
11. Suppose  is continous on [a,b], differentiable on (a,b) and satisfies .

Show that the equation  has at least one root in (a,b).

1. Show that the function is differentiable at 0. More generally, if is continuous at 0, then is differentiable at 0. Examine the function for differentiability.
2. Show that the equation has exactly one (real) root.
3. Let and be functions, continuous on , differentiability on  and let .Prove that there is a point such that .
4. Let be differentiable. Then is constant if and if for every .
5. What do you think about the supremum and infimum of empty set?
6. If and if for all , prove that is a decreasing sequence with limit 0.
7. Find the sup and inf of .
8. Given two non-empty subsets A and B of such that sup A = a and sup B = b. Now we define C=. Show that sup C = a + b.
9. Show that .
10. If is a non empty subset of such that sup = inf. Then what can you say about
11. What will happen if we take open intervals instead of closed intervals in Nested Interval Theorem? Will theorem be still true?
12. Prove that every convergent sequence is bounded, but converse is not true.
13. Show that if a sequence is monotonic and bounded then it is convergent.
14. Let  and . For a positive integer k show that



1. Let {} be the sequence of strictly positive real numbers such that . Then

Prove the following.

1. If  < 1 then ,
2. If  > 1 then 
3. What will happen if = 1.
4. Investigate the convergence of the sequence

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